MU ALPHA THETA - 2012
ALPHA INDIVIDUAL TEST

1. Factor $a^2 + b^2 - 2ab - 9c^2$

2. Find the radius of $x^2 + y^2 - 8x + 4y - 8 = 0$ (answer exactly)

3. What is the remainder when $8x^4 - 5x^3 - 39x - 4$ is divided by $x - 2$?

4. If $\sin x = -\frac{2}{3}$ and $\tan x = -\frac{2\sqrt{5}}{5}$, find $\cos x$

5. Solve $\sqrt{x+6} - \sqrt{x-1} = 1$

6. Find the range of $f(x) = -2x^2 + 24x - 61$

7. Simplify $\cos\left(\frac{\pi}{6} + x\right) + \cos\left(\frac{\pi}{6} - x\right)$

8. An exponential function has y intercept 16 and $f(3) = \frac{27}{4}$.
   Find $f(-1)$

9. Solve $\sqrt{\frac{9^{x+3}}{27^x}} = 81$

10. Find x if $\log_{15}\left(\log_5\left(\log_2 x\right)\right) = 0$

11. Find $\cos(22\frac{1}{2}^\circ)$(answer exactly)

12. Find all values of k for which $2x^2 - kx + x + 8 = 0$ has real and equal roots.

13. Simplify $(\sec x + \tan x)(1 - \sin x)$

14. In ΔABC, $a = 24$, $b = 30$ and $m\angle C = 30°$ Find the area of ΔABC.

15. If $f(x) = x^3 + 4x + 1$, find $f^{-1}(17)$

16. Find the equation of an ellipse having center $(3,7)$, a focus at $(6,7)$ and a vertex at $(8,7)$

17. Solve $2x^{2/3} - 5x^{1/3} - 12 = 0$

18. Solve: $7x^2 + 20x - 3 < 0$ (answer exactly)

19. Simplify $i^{25} + i^{36} + i^{47} + i^{58}$

20. A lighthouse keeper observes that there is a 3˚ angle of depression between the horizontal and the line of sight to a ship. If the keeper is 19 m above the water, how far is the ship from shore (to the nearest meter)?

2012 ALPHA INDIVIDUAL TEST SOLUTIONS

1. $(a-b+3c)(a-b-3c)$

2. $2\sqrt{7}$

3. 6

4. $\frac{\sqrt{5}}{3}$

5. $x=10$

6. $(-\infty,11]$ or $y \le 11$

7. $\sqrt{3}\cos x$

8. $\frac{64}{3}$ or $21.\overline{3}$

9. $x=-2$

10. 32

11. $\frac{\sqrt{2+\sqrt{2}}}{2}$

12. $k=9,\ -7$

13. $\cos x$

14. 180

15. 2

16. $\frac{(x-3)^2}{25}+\frac{(y-7)^2}{16}=1$

17. $-\frac{27}{8},\ 64$

18. $\left(-3,\frac{1}{7}\right)$ or $-1<x<\frac{1}{7}$

19. 0

20. 363 m