

MU ALPHA THETA - 2012  
MU A INDIVIDUAL TEST

1. Find  $\lim_{x \rightarrow 0} \left( \frac{e^x - 1}{e^{2x} - 1} \right)$

2. Find  $\lim_{x \rightarrow -\infty} \left( \frac{\sqrt{2 + 3x^2}}{x + 4} \right)$

3. Find  $\lim_{h \rightarrow 0} \frac{(5 + h)^4 - 5^4}{h}$

4. If  $g(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$  find  $g'''(4)$

5. On what open interval(s) is  $f(x) = x^3 - x^2 - x + 1$  decreasing?

6. Find the slope of the line tangent to  $x^3y + xy^3 = 2$  at  $(1,1)$
7. If  $f(x) = x^{2/3}$ , find a value of  $x$  on  $[0,1]$  that satisfies the Mean Value Theorem
8. A particle's position is given by  $s(t) = \frac{t}{t^2 + 1}$  ( $s$  in feet,  $t$  in seconds).  
Find the velocity of the particle at 2 seconds (include units)
9. Find all  $x$  values for which  $f(x) = (2x + 1)^2(x - 3)^4$  has a horizontal tangent
10. Air is being pumped into a spherical balloon in such a way that its volume is increasing at a constant rate of  $200\pi$  cm<sup>3</sup>/s. Find the rate of change of the radius when the radius is 5 cm. (include units)

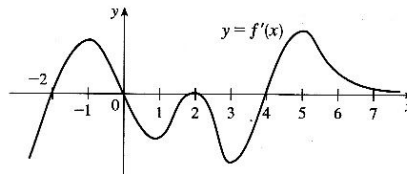
11. On what open interval(s) is  $f(x) = xe^x$  concave up?

12. If  $y = \cos^2(\tan x)$  find  $\frac{dy}{dx}$

13. Find the minimum function value for  $f(x) = x - \sqrt{1-x^2}$  on  $[-1, 1]$  (answer exactly)

14. Find  $\frac{d}{dx} \int_e^{x^2} \ln x dx$

15. The figure shows the graph of the derivative  $f'$  of a function. For what value(s) of  $x$  does  $f$  have a local extreme value?



16. Given the values in the table below, if  $h(x) = f(g(x))$ , find  $h'(2)$

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	3	-2	5	4
5	28	11	26	10

17. Find  $\int_{-3}^1 f(x) dx$  for  $f(x) = \begin{cases} -x-1 & -3 \leq x \leq 0 \\ -\sqrt{1-x^2} & 0 < x \leq 1 \end{cases}$  (answer exactly)

18. Find  $\int \tan x \ln(\cos x) dx$

19. Find the average value of  $f(x) = \frac{1}{x}$  on  $[1, 3]$  (answer exactly)

20. Find the area enclosed by  $y = e^x$ ,  $y = e^{3x}$  and  $x = 1$  (answer exactly)

2012 MU A INDIVIDUAL TEST SOLUTIONS

1.  $\frac{1}{2}$

2.  $-\sqrt{3}$

3. 500

4.  $-\frac{3}{1024}$

5.  $\left(-\frac{1}{3}, 1\right)$

6. -1

7.  $\frac{8}{27}$

8.  $-\frac{3}{25}$  ft/sec (must have units)

9.  $x = -\frac{1}{2}, \frac{2}{3}, 3$

10. 2 cm/s

11.  $(-2, \infty)$

12.  $-\sin(2 \tan x) \sec^2 x$  or  $-2 \sin(\tan x) \cos(\tan x) \sec^2 x$

13.  $-\sqrt{2}$

14.  $2x \ln(x^2)$  or  $4x \ln x$

15. -2, 0, 4

16. 44

17.  $\frac{3}{2} - \frac{\pi}{4}$  or  $\frac{6 - \pi}{4}$

18.  $-\frac{1}{2}(\ln(\cos x))^2 + C$

19.  $\frac{1}{2} \ln 3$  or  $\ln \sqrt{3}$

20.  $\frac{1}{3} e^3 - e + \frac{2}{3}$