

Mu Alpha Theta 2012

Functions

Select the correct answer for each problem. If a correct answer is not given, choose the choice NOTA (none of the above).

1. Which of the following are functions?

I. $y = x^3 + 2x^2 + 5$

II. $\sqrt{y} = x$

III. $|y| = (x+1)^2$

IV. $x = \log y$

A. I only

B. I, II

C. I, IV

D. all

E. NOTA

2. Let $f(n)$ represent the sum of the first n positive integers $\left[f(n) = \left(\frac{n}{2}\right)(n+1) \right]$. Solve the equation below for the positive value of n .

$$f(4) + f(8) + 9 = f(n)$$

A. 9

B. 10

C. 11

D. 12

E. NOTA

3. If $f(x) = (x+3)^2$ and $x \leq -3$, then find $f^{-1}(x)$.

A. $\sqrt{x} - 3$

B. $\sqrt{x} + 3$

C. $-\sqrt{x} - 3$

D. No Inverse

E. NOTA

4. Find all the asymptotes of the graph: $f(x) = \frac{x-3}{x^2-x-6}$

A. $y = 0, x = 3, x = -2$

B. $y = 1, x = 3, x = -2$

C. $y = 0, x = -2$

D. $y = 1, x = -2$

E. NOTA

5. A football player kicks off and the height of the football is determined by the function

$h(t) = -\frac{25t^2}{2} + 50t$, where t is time. How long is the football in the air before it hits the ground?

A. $\frac{3}{2}$

B. 2

C. $\frac{5}{2}$

D. 4

E. NOTA

6. Let $f(x) = x^5 - 3x^4 + 4x^3 - 4x^2 - 5x + 1$. How many negative roots does $f(x)$ have?
 A. 0 B. 1 C. 2 D. 3 E. NOTA
7. The Mobius function is defined over integers as follows:

$$\mu(x) = \begin{cases} -1 & \text{if } x \text{ is the product of an odd number of distinct primes} \\ 1 & \text{if } x \text{ is the product of an even number of distinct primes} \\ 0 & \text{if } x \text{ contains any multiple prime factors} \end{cases}$$
 Determine $\mu(26)$.
 A. -1 B. 0 C. 1 D. 26 E. NOTA
8. Find the range of the function $f(x) = x^2 - 2x + 3$.
 A. $(-\infty, -2]$ B. $(-\infty, 2]$ C. $[-2, \infty)$ D. $[2, \infty)$ E. NOTA
9. Given: $f(x+2) = 3x^3 - 2x^2 - 4x - 1$, find $f(3)$.
 A. -4 B. -2 C. 2 D. 50 E. NOTA
10. Given $f(x) = ax$, $g(x) = bx$, $h(x) = x + 1$, and $f(g(h(x))) = h(g(f(x)))$, find the value of ab . (Let $a \neq 0$ and $b \neq 0$)
 A. 0 B. 1 C. Set of all \Re D. x E. NOTA
11. Define $f(x) = \sqrt{x+2}$ and $g(x) = x^2$. If $(f \circ g \circ h)(x) = \sqrt{x^2 + 2x + 3}$, then which of the following are possible definitions of $h(x)$?
 A. $x + 1$ B. $x - 1$ C. x D. 1 E. NOTA
12. Suppose $h(x)$ which is not everywhere zero, has a well-defined inverse function $h^{-1}(x)$ that satisfies $(h^{-1} \circ h)(x) = x$ for all x in the domain of h . Which of the following must **always** be false?
 A. $h(x)$ is even B. $h(x)$ is odd C. $h(x)$ is one-to-one
 D. $h(0) = 1$ E. NOTA
13. Given the function $f(x) = 2x^3 - 3x^2 - kx + 5$. If the sum of the reciprocals of the roots is 2 times larger than the sum of the roots, then find the value of k .
 A. -15 B. $-\frac{15}{4}$ C. $\frac{15}{4}$ D. 15 E. NOTA

14. Define $f(x) = x^2$ and $g(x) = -x^3$ and $h(x) = (f \circ g + g \circ f)(x)$. Which of the following are true?
 I. $h(x)$ is an odd function II. $h(x)$ is an even function
 A. I only B. II only C. both I and II D. neither I nor II E. NOTA
15. Let $h(x) = \log(x^2 + 2x + 2)$. Which of the following is a root of $h(x)$?
 A. 1 B. $\frac{3}{2}$ C. 2 D. $\frac{5}{2}$ E. NOTA
16. What are the domain restrictions on $f(x) = \sqrt{\frac{x-1}{x+1}}$, so that $f(x)$ is real-valued?
 A. $-1 < x < 1$ B. $x \leq -1$ or $x > 1$ C. $-1 < x < 1$ D. $x < -1$ or $x \geq 1$ E. NOTA
17. Find the vertical asymptotes of $y = (g \circ f)(x)$ when $f(x) = \frac{x+1}{x+2}$ and $g(x) = \frac{1}{x(x-2)}$.
 A. $x = 0, x = 2$ B. $x = -1, x = 1$ C. $x = 0, x = -2$ D. $x = -1, x = -3$ E. NOTA
18. The graphs of $f(x)$ and $f^{-1}(x)$ are reflections of each other over which line?
 A. $x = 0$ B. $y = 0$ C. $y = x$ D. $y = -x$ E. NOTA
19. Given that $f(x)$ and $f^{-1}(x)$ are both defined and $f(x) = \frac{2x}{3x-8}$, find $f^{-1}(x)$ where defined.
 A. $\frac{-8x}{3x-2}$ B. $\frac{-8x}{3x+2}$ C. $\frac{8x}{3x-2}$ D. $\frac{8x}{3x+2}$ E. NOTA
20. Suppose $p(x)$ is a fifth degree real-valued polynomial. If $1+i$ and $2-i$ are roots, how many real roots does $p(x)$ have?
 A. 0 B. 1 C. 2 D. 3 E. NOTA
21. Let $f(x)$ be a linear function with a positive slope and $f(x) = 4f^{-1}(x) - 5$. What is $f(2)$?
 A. -6 B. -1 C. $\frac{7}{3}$ D. 2 E. NOTA

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Answer Key

1. E
2. B
3. C
4. C
5. D
6. B
7. C
8. D
9. A
10. B
11. A
12. A
13. D
14. C
15. E
16. D
17. D
18. C
19. C
20. B
21. C
22. C
23. A
24. C
25. B