

LOUISIANA MU ALPHA THETA STATE CONVENTION
SCHOLARSHIP TEST 2012

1. A triangle has area 21 cm^2 and two of its sides are 9 cm and 14 cm long. Find the possible measures of the angle formed by these sides.
2. Solve: $3x^4 - 18x^3 + 27x^2 - 6x = 0$ (answer exactly)
3. A complex number is represented by $4cis \frac{7\pi}{4}$. Express it in $a + bi$ form.
4. Which term of the arithmetic sequence $-1, 5, 11, 17, \dots$ is 245?
5. Find the equation of a parabola whose focus is $(4, 2)$ and whose directrix is $x = 6$.
6. If $f(x) = x^3 + 4x + 1$, find $f^{-1}(6)$
7. Three laser printers can print out the same job in 20 minutes, 30 minutes, and 60 minutes, respectively. How long would it take to finish the job if all three printers are running?
8. Simplify: $\sin\left(A - \frac{\pi}{3}\right) \cos\left(A + \frac{\pi}{3}\right) - \cos\left(A - \frac{\pi}{3}\right) \sin\left(A + \frac{\pi}{3}\right)$
9. An object is dropped from 500 feet. Use the Mean Value Theorem to find the time during the first three seconds when the instantaneous velocity equals the average velocity.
10. Solve: $\log_2(x^2 + 1) - \log_4(x^2) = 1$

11. Solve $2 \cos^2 x - \cos x = 2 - \sec x$ on $[0, 2\pi)$
12. For which values of x is $\frac{1}{x + \frac{1}{x+3}}$ undefined? (answer exactly)
13. A revolving searchlight in a lighthouse 2 miles offshore is following a beachcomber along the shore. When the beachcomber is 1 mile from the point on the shore that is closest to the lighthouse, the searchlight is turning at a rate of 0.25 rev/hour. How fast is the beachcomber walking at that moment?
14. If $\sin x = \frac{4}{5}$ $\left(0 < x < \frac{\pi}{2}\right)$ and $\sin y = \frac{24}{25}$ $\left(\frac{\pi}{2} < y < \pi\right)$, find $\cos(x+y)$. (answer exactly)
15. Simplify $\lim_{h \rightarrow 0} \frac{\log_2(h+5) - \log_2 h}{h}$ (answer exactly)
16. Expand, then evaluate $\sum_{i=1}^5 (2^i - 1)$
17. Find a function f and the smallest positive constant a such that $2 \int_a^x f(t) dt = 2 \sin x - 1$
18. Find the x value(s) of the points on the graph of $x^3 - y^3 = 3xy - 3$ where the tangent line is horizontal.
19. Given that $12a + 10b = 1020$, what is the value of $\frac{a}{5} + \frac{b}{6}$?
20. Find A and B if $\frac{x-7}{x^2+x-6} = \frac{A}{x+3} + \frac{B}{x-2}$

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1. 19.5° or 20.5°
2. $0, 2, 2 \pm \sqrt{3}$
3. $2\sqrt{2} - 2\sqrt{2}i$
4. 42
5. $x - 5 = -\frac{1}{4}(y - 2)^2$
6. 1
7. 10 minutes
8. $-\frac{\sqrt{3}}{2}$
9. 1.5 seconds
10. $x = 1$
11. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$
12. $-3, \frac{-3 \pm \sqrt{5}}{2}$
13. 3.927 mi/h
14. $-\frac{117}{125}$
15. $\frac{1}{5 \ln 2}$
16. 57
17. $f(x) = \cos x, a = \frac{\pi}{6}$
18. $x = 1, -\sqrt[3]{3}$
19. 17
20. $A = 2, B = -1$